

# C.U.SHAH UNIVERSITY

## Summer Examination-2017

**Subject Name : Engineering Mathematics-I**

**Subject Code : 4TE01EMT1**

**Branch: B.Tech(All)**

**Semester : 1**

**Date :22/03/2017**

**Time :10:30 To 01:30**

**Marks : 70**

**Instructions:**

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q.1 Attempt the following questions: (14)**

**A) If the power of  $x$  &  $y$  both are even. then the curve is symmetrical about (1)**

- (a) X-axis      (b) Y-axis      (c) both X & Y axes      (d) none of these

**B) If  $y = \sin^{-1} x$ , then  $x$  equal to (1)**

- (a)  $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$       (b)  $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$   
 (c)  $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$       (d) none of these

**C) Define Euler's Theorem. (1)**

**D) If  $x = r \cos \theta$  &  $y = r \sin \theta$ , then  $\frac{\partial(x, y)}{\partial(r, \theta)} =$  \_\_\_\_\_ (1)**

- (a) 0      (b) 1      (c)  $\frac{1}{r}$       (d) r

**E)  $\lim_{x \rightarrow 0} \cos x =$  \_\_\_\_\_. (1)**

- (a) 0      (b) 1      (c)  $\infty$       (d) -1

**F)  $e^{10\pi i} =$  \_\_\_\_\_ (1)**

- (a) 0      (b) 1      (c) -1      (d) None of these

**G) Find  $\frac{dy}{dx}$  for  $x^2 + y^3 = 7xy$ . (1)**

**H) If the two tangents at the point are real & distinct, the double point is called (1)**

- (a) a node      (b) a cusp      (c) a conjugate point      (d) None of these

**I) The series  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$  represents expansion of (1)**



- (a)  $\sin x$       (b)  $\cos x$       (c)  $\cosh x$       (d)  $\sinh x$

J) The series  $\sum \frac{1}{n}$  is (1)

- (a) Convergent    b) Divergent    c) non-convergent    d) a & b both

K)  $\lim_{(x,y) \rightarrow (2,2)} \frac{3x+4y}{x+y} = \underline{\hspace{2cm}}$ . (1)

- (a) 0      (b) 1      (c)  $\frac{7}{2}$       (d)  $\frac{3}{2}$

L) What is the argument of  $z = 1 + i$  (1)

- (a)  $-\frac{3\pi}{4}$       (b)  $\frac{3\pi}{4}$       (c)  $\frac{\pi}{4}$       (d)  $-\frac{\pi}{4}$

M)  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x} = \underline{\hspace{2cm}}$ . (1)

- (a) 0      (b) 1      (c)  $\infty$       d)-1

N) If  $J = \frac{\partial(u, v)}{\partial(x, y)}$  &  $J' = \frac{\partial(x, y)}{\partial(u, v)}$  then  $JJ' = \dots\dots\dots$  (1)

- (a) 1      (b) -1      (c) 0      (d) None of these

**Attempt any four questions from Q-2 to Q-8**

**Q.2 Attempt all questions** (14)

A) Trace the curve (Cardioid)  $r = a(1 + \cos \theta)$ . (07)

B) Discuss the convergence of  $\sum_{n=1}^{\infty} \left( \frac{\sqrt{n+1} - \sqrt{n}}{n} \right)$  (04)

C) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1^x + 2^x + 3^x}{3} \right)^{\frac{1}{x}}$ . (03)

**Q.3 Attempt all questions** (14)

A) Test the Convergence of the series  $\sum_{n=1}^{\infty} (ne^{-n^2})$ . (05)

B) If  $u = \ln(x^3 + y^3 + z^3 - 3xyz)$  then prove that (05)

$$\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}$$

C) Show that  $\frac{(\cos 2\theta - i \sin 2\theta)^5 (\cos 3\theta + i \sin 3\theta)^6}{(\cos 4\theta + i \sin 4\theta)^7 (\cos \theta - i \sin \theta)^8} = \cos 12\theta - i \sin 12\theta$ . (04)

**Q.4 Attempt all questions** (14)

A) Test the series for absolute or conditional convergence (05)

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots\dots$$



- B) Find the Taylor's expansion of  $\tan\left(x + \frac{\pi}{4}\right)$  in ascending powers of  $x$  up to  $x^4$  & find approximate value of  $\tan(43^\circ)$ . (05)

- C) Discuss the continuity of  $f(x, y) = \begin{cases} \left(\frac{x^3 - y^3}{x^2 + y^2}\right); (x, y) \neq (0, 0) \\ 0; (x, y) = (0, 0) \end{cases}$  (04)

**Q.5 Attempt all questions** (14)

- A) Trace the curve (Cisoid of Diocle)  $y^2(2a - x) = x^3$ . (07)
- B) Find & plot all roots of  $\sqrt[3]{8i}$ . (04)
- C) Evaluate:  $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\tan^2 x}\right)$ . (03)

**Q.6 Attempt all questions** (14)

- A) State Modified Euler's Theorem. If  $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ , then (05)

show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$

- B) Prove that  $\cos^8 \theta = \frac{1}{128} [\cos 8\theta + 8 \cos 6\theta + 28 \cos 4\theta + 56 \cos 2\theta + 35]$  (05)
- C) Define  $\log(x + iy)$  and Show that the set of  $\log(i^2)$  is not same as the set of values of  $2 \log(i)$ . (04)

**Q.7 Attempt all questions** (14)

- A) Find maxima & minima of the Function  $f(x, y) = x^2 y - xy^2 + 4xy - 4x^2 - 4y^2$ . (05)
- B) Obtain the range of convergence  $\sum_{n=1}^{\infty} \left(\frac{x^n}{2^n}\right), x > 0$ . (05)
- C) Find  $\frac{dy}{dx}$  if  $y^{x^y} = \sin x$  (04)

**Q.8 Attempt all questions** (14)

- A) Find the radius of convergence & interval of convergence of the series  $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ . (05)
- B) Define Errors. Find the percentage error in calculating the area of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , when error of 1% is made in measuring its major & find minor axes. (05)
- C) If  $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$ , find  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ . (04)

